

CHAPTER 10 다변수함수

SECTION 10.1 다변수함수

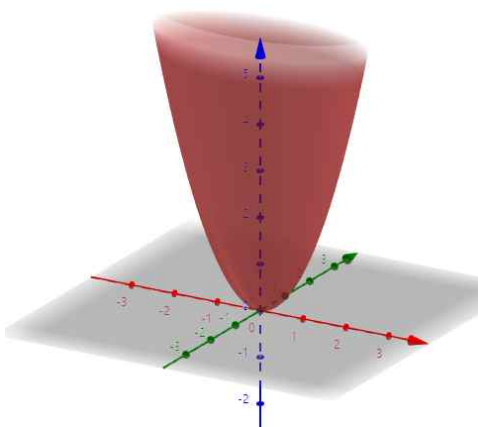
유제 10-1

(a) 정의역: $y \neq \frac{5}{2}x, y \leq 3x$ (x, y 는 실수)

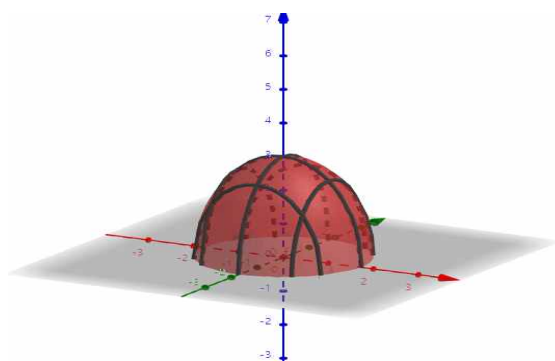
(b) 정의역 : $\frac{x}{y} > 2, y \neq 0$

유제 10-2

(a)

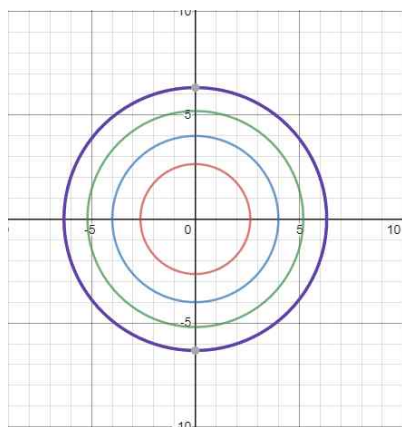


(b)



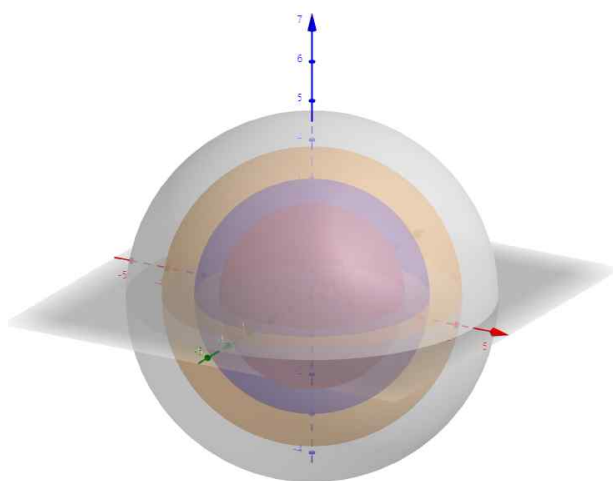
유제 10-3

등위곡선 $\sqrt{9+x^2+y^2}=k$



유제 10-4

등위곡선 $\sqrt{x^2+y^2+z^2-4}=k$



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10.1 연습문제

01

$$\{(x, y) \in \mathbf{R}^2 \mid x \geq 1, y \in \mathbf{R}\}$$

02

$$\{(x, y) \in \mathbf{R}^2 \mid x + y < 1\}$$

03

$$\{(x, y) \in \mathbf{R}^2 \mid x^2 + y^2 \geq 9\}$$

04

$$\{(x, y) \in \mathbf{R}^2 \mid y \neq x^2\}$$

05

$$\{(x, y) \in \mathbf{R}^2 \mid x > 0, y \neq 0\}$$

06

$$\{(x, y) \in \mathbf{R}^2 \mid |x| > |y| > 0\}$$

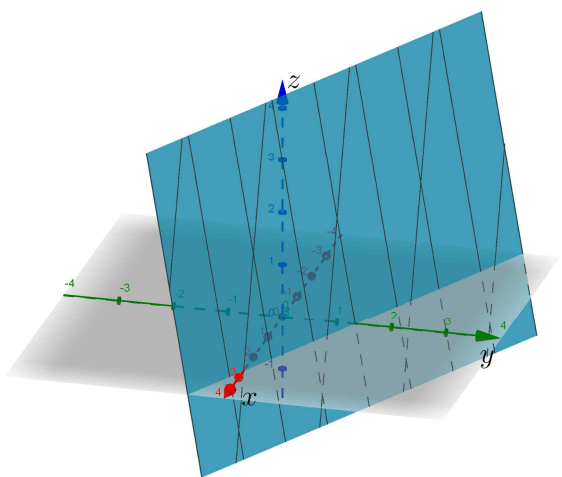
07

$$\{(x, y, z) \in \mathbf{R}^3 \mid x^2 + y^2 + z^2 \leq 8\}$$

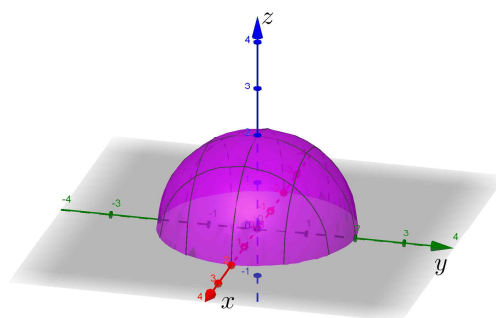
08

$$\{(x, y, z) \in \mathbf{R}^3 \mid x > y \circledast z \mid z \in \mathbf{R}\}$$

09

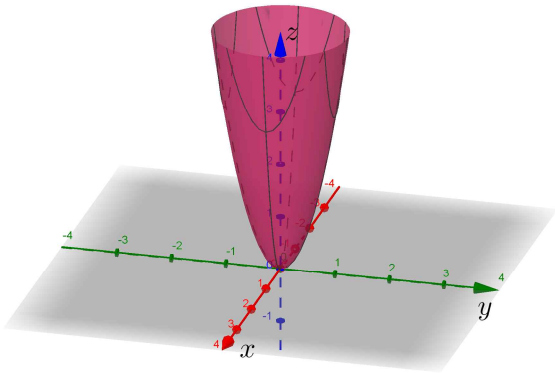


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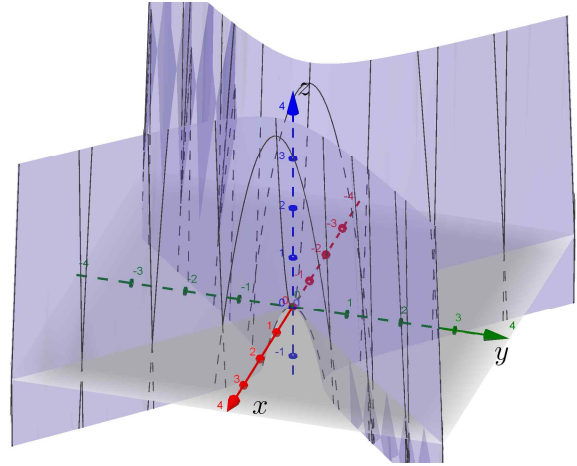


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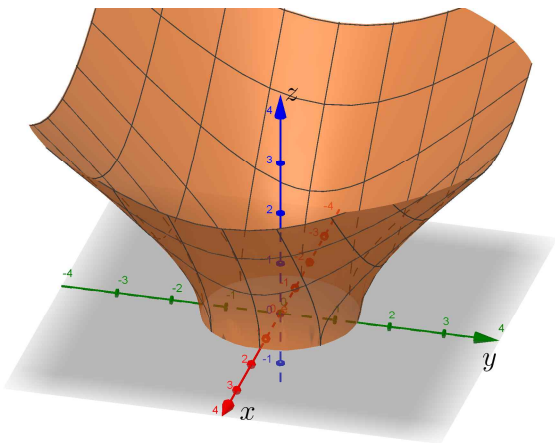
11



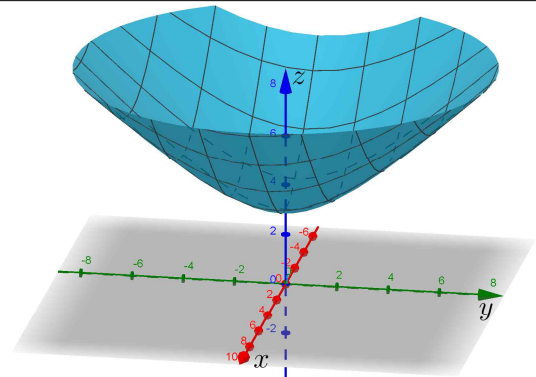
12



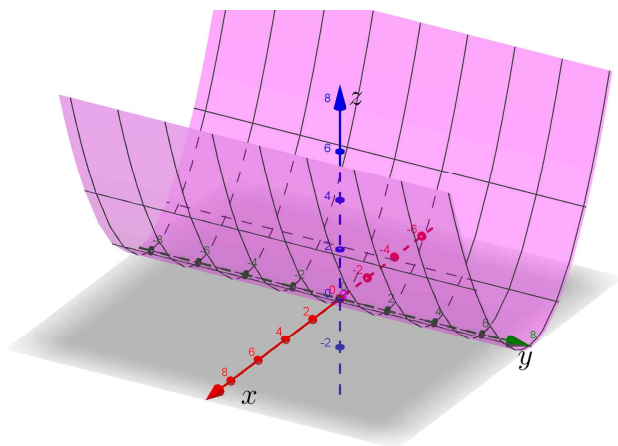
13



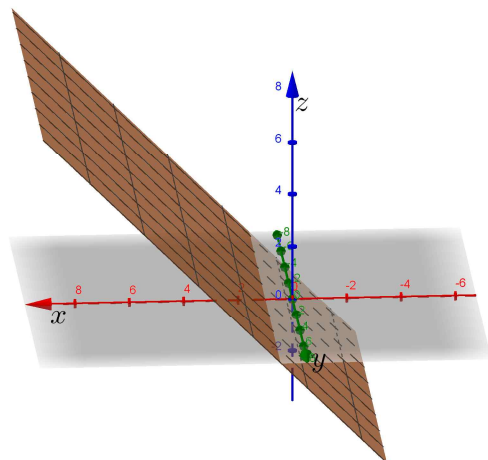
14



15



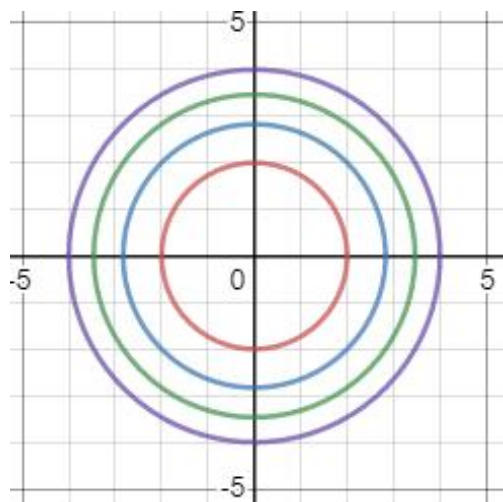
16



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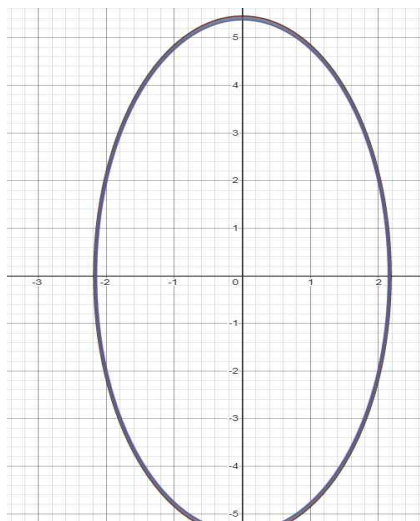
17

등위곡선 : $\frac{1}{4}(x^2 + y^2) = k$



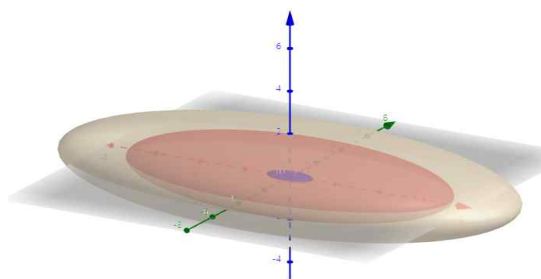
18

등위곡선 $120 - 25x^2 - 4y^2 = k$



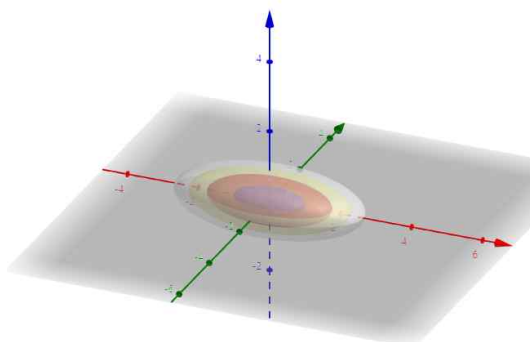
19

등위곡선 $\sqrt{150 - x^2 - 4y^2 - 25z^2} = k$



20

등위곡선 $x^2 + 4y^2 + 9z^2 = k$



CHAPTER 10 다변수함수

SECTION 10.2 다변수함수의 극한과 연속

유제 10-5

$$\frac{6}{7}$$

유제 10-6

중심이 $(0,0)$ 이고 반지름이 3인 원에서 불연속

유제 10-7

1

10.2 연습문제

01

4

02

$$\frac{\sqrt{2}}{4}$$

03

$$\frac{16}{3}$$

04

$$e^3$$

05

0

06

0

07

2

08

1

09

$$-\frac{1}{2}$$

10

1

11

존재하지 않는다.

12

존재하지 않는다.

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13

0

14

존재하지 않는다.

15

존재하지 않는다.

16

존재하지 않는다.

17

모든 실수 x, y 에 대하여 연속

18

$x + y^2 = 1$ 인 점 (x, y) 에서 불연속

19

모든 실수 x, y 에 대하여 연속

20

모든 실수 x, y 에 대하여 연속

21

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22

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SECTION 10.3 편도함수

유제 10-8

- (a) -8
- (b) -525

유제 10-9

- (a) $x + 3y$
- (b) $3x - 3y^2$
- (c) 4
- (d) -12

유제 10-10

$$f_x = \frac{4x}{y^3 + 3z}, \quad f_y = -\frac{6x^2 y^2}{(y^3 + 3z)^2}, \quad f_z = -\frac{6x^2}{(y^3 + 3z)^2}$$

유제 10-11

$$f_{xy}(x, y, z) = (3 + 3xy - 3z)e^{xy} \cos(x + yz) + (-3yz - 3x)e^{xy} \sin(x + yz)$$

$$f_{yz}(x, y, z) = -3xye^{xy} \sin(x + yz)$$

$$f_{zx}(x, y, z) = -3y^2 e^{xy} \sin(x + yz) - 3ye^{xy} \cos(x + yz)$$

10.3 연습문제

01

$$\frac{\partial z}{\partial x} = 2 + 15x^2 y^2, \quad \frac{\partial z}{\partial y} = -28y^3 + 10x^3 y$$

02

$$\frac{\partial z}{\partial x} = 3y^2 e^{xy^2}, \quad \frac{\partial z}{\partial y} = 6xy e^{xy^2}$$

03

$$\frac{\partial z}{\partial x} = \ln(1 + x^3 y^{-5/2}) + \frac{3x^3 y^{-5/2}}{1 + x^3 y^{-5/2}}, \quad \frac{\partial z}{\partial y} = -\frac{5}{2} \cdot \frac{x^4 y^{-7/2}}{1 + x^3 y^{-5/2}}$$

04

$$\frac{\partial z}{\partial x} = 2xy e^{x^2 y} \cos 2y^3, \quad \frac{\partial z}{\partial y} = x^2 e^{x^2 y} \cos 2y^3 - 6y^2 e^{x^2 y} \sin 2y^3$$

05

$$2e$$

06

$$3e$$

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07

$$-\frac{\pi}{2}$$

08

$$-\frac{1}{4}$$

09

-1 (2021-07-21 updated)

10

$$f_{xx} = e^x \cos y, f_{yy} = -e^x \cos y, f_{xy} = -e^x \sin y, f_{yx} = -e^x \sin y$$

11

$$f_{xx} = -\frac{25}{(5x+2y)^2}, f_{yy} = -\frac{4}{(5x+2y)^2}, f_{xy} = -\frac{10}{(5x+2y)^2}, f_{yx} = -\frac{10}{(5x+2y)^2}$$

12

$$f_{xxy} = 30xy^4 - 4$$

13

$$f_{yxy} = 60x^2y^3$$

14

$$f_{yyy} = 60x^3y^2$$

15

$$-60$$

16

-15 (2021-07-21 updated)

17

$$-30$$

18

$$f_x(0,0) = -1$$

$$f_y(0,0) = -2$$

19

$$f_x(0,y) = -2, f_y(x,0) = -3$$

20

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SECTION 10.4 연쇄법칙

유제 10-12

$$(a) \quad \frac{dw}{dt} = (t^2 \cos(e^t t^2) + \frac{t^2}{e^t})e^t + (e^t \cos(e^t t^2) + t)2t$$

$$(b) \quad \frac{dw}{dt} = \frac{1}{2\sqrt{\sin t} \, 3t - e^{\sin t}} (3t - e^{\sin t}) \cos t + \frac{1}{2\sqrt{\sin t} \, 3t - e^{\sin t}} 3 \sin t$$

유제 10-13

$$\frac{\partial z}{\partial u} = 24u^2v^2 - 16uv^3 - 2v + 3$$

$$\frac{\partial z}{\partial v} = 16u^3v - 24u^2v^2 - 2u - 3$$

유제 10-14

$$\frac{\partial f}{\partial u} = (2xy \cos x^2 y)(v - 2w \sin uw) + (x^2 \cos x^2 y + ze^{yz})(4u - e^{u+v}) + (ye^{yz} + 2z)(\frac{1}{u+2w})$$

$$\frac{\partial f}{\partial v} = u(2xy \cos x^2 y) - e^{u+v}(x^2 \cos x^2 y + ze^{yz}) + 3(ye^{yz} + 2z)$$

$$\frac{\partial f}{\partial w} = (2xy \cos x^2 y)(3 - 2u \sin uw) + x^2 \cos x^2 y + ze^{yz} + (\frac{2}{u+2w} - 1)(ye^{yz} + 2z)$$

유제 10-15

$$\frac{dy}{dx} = \frac{-3x^2 - 4xy^2 + 1}{4x^2y - 12y^3 - 2}$$

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10.4 연습문제

01

$$16t^{13} + 12t^{13} = 28t^{13}$$

02

$$\frac{4\sqrt{t}}{2t + t^{\frac{1}{3}}} \cdot \frac{1}{2\sqrt{t}} + \frac{1}{2t + t^{\frac{1}{3}}} \cdot \frac{2}{3\sqrt[3]{t}}$$

03

$$\frac{3}{t^2} \sin\left(\frac{1}{t}\right)$$

04

$$\frac{1 - 2t^4}{2\sqrt{1 + \ln t - 2(\ln t)t^4}} \cdot \frac{1}{t} + \frac{-8(\ln t)t^3}{2\sqrt{1 + \ln t - 2(\ln t)t^4}}$$

05

$$-\frac{10}{3}t^{\frac{7}{3}}e^{1-t^{\frac{10}{3}}}$$

06

$$e^t(1+t)\left(\frac{1}{4}(e^{te^t} - e^{-te^t})\right)$$

07

$$\frac{\partial z}{\partial u} = (16uv(u-v) - 2)v + (8u^2v^2 + 3)$$

$$\frac{\partial z}{\partial v} = (16uv(u-v) - 2)u - (8u^2v^2 + 3)$$

08

$$\frac{\partial z}{\partial u} = \left(\frac{2u}{v} - u^2v^2 \sec^2\left(\frac{u}{v}\right)\right) \frac{1}{v} - \tan\left(\frac{u}{v}\right) 2uv^2$$

$$\frac{\partial z}{\partial v} = \left(\frac{2u}{v} - u^2v^2 \sec^2\left(\frac{u}{v}\right)\right) \frac{-u}{v^2} - \tan\left(\frac{u}{v}\right) 2u^2v$$

09

$$\frac{\partial z}{\partial u} = -\frac{2\sin u}{3\sin v}$$

$$\frac{\partial z}{\partial v} = -\frac{2\cos u \cos v}{3\sin^2 v}$$

10

$$\frac{\partial z}{\partial u} = 3\left(1 + \frac{v}{u}\right) - 4u$$

$$\frac{\partial z}{\partial v} = 3\ln u + 2\ln v + 2$$

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11

$$\frac{\partial z}{\partial u} = e^u$$

$$\frac{\partial z}{\partial v} = 0$$

12

$$\frac{\partial z}{\partial u} = -\sin(u-v)\sin(u^2+v^2) + (\cos(u-v)\cos(u^2+v^2))2u$$

$$\frac{\partial z}{\partial v} = \sin(u-v)\sin(u^2+v^2) + \cos(u-v)\cos(u^2+v^2)2v$$

13

1161

14

$$-\pi$$

15

$$\left. \frac{\partial f}{\partial u} \right|_{u=1, v=-2} = \frac{351}{2}$$

$$\left. \frac{\partial f}{\partial v} \right|_{u=1, v=-2} = -168$$

16

$$\left. \frac{\partial z}{\partial r} \right|_{r=2, \theta=\frac{\pi}{6}} = \sqrt{3}e^{\sqrt{3}}$$

$$\left. \frac{\partial z}{\partial \theta} \right|_{r=2, \theta=\frac{\pi}{3}} = (-2 - \frac{4}{3}\sqrt{3})e^{\frac{1}{\sqrt{3}}}$$

17

증명 생략

18

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19

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CHAPTER 10 다변수함수

SECTION 10.5 방향도함수와 기울기벡터

유제 10-16

$$\frac{3}{\sqrt{5}}(2e^7 + 1)$$

유제 10-17

$$\frac{1}{2}y \cos(xy) + \frac{\sqrt{3}}{2}x \cos(xy) + 1 + \frac{\sqrt{3}}{2}$$

유제 10-18

$$\frac{4}{\sqrt{5}x} + \frac{2}{\sqrt{5}y} + \frac{6}{\sqrt{5}}y + \frac{2}{\sqrt{5}}$$

유제 10-19

(a) (1341, 1647)

(b) $\frac{1341}{2} + \frac{1647\sqrt{3}}{2}$

유제 10-20

$$\frac{4}{\sqrt{6}} - \frac{1}{\sqrt{6}}e^2 - \frac{2}{\sqrt{6}}e$$

유제 10-21

등위곡선 : $\frac{x^2}{9} + 4y^2 = 17$

기울기벡터 : $(\frac{2}{3}, 16)$ (2021-07-21 updated)

10.5 연습문제

01

$$D_u f = 160 - 128\sqrt{3}$$

02

$$D_u f = \frac{2}{5}(4e^2 + \frac{1}{3}) - \frac{\sqrt{21}}{5}(e^2 + \frac{1}{3})$$

03

$$D_u f = -\frac{314}{325}$$

04

CHAPTER 10 다변수함수

$$D_u f = \frac{-1}{12} \pi + \frac{1}{12}$$

05

$$D_u f = \frac{24}{\sqrt{10}} xy^2 + \frac{12}{\sqrt{10}} x^2 y^2$$

06

$$D_u f = \frac{1}{\sqrt{2} x} - \frac{3}{\sqrt{2}} y^2$$

07

$$D_u f = = \frac{6}{7(z+y)} + \frac{x-2y-3z}{7(z+y)^2}$$

08

$$D_u f = \frac{31}{21} e^{x+y+3z}$$

09

$$u = \frac{1}{\sqrt{85}} (6, 7)$$

10

$$u = \frac{4}{\sqrt{41}} \left(\frac{1}{\sqrt{32}}, \frac{9}{\sqrt{32}} \right)$$

11

$$u = \frac{1}{\sqrt{16 + \frac{1}{e^2}}} \left(4, \frac{1}{e} \right)$$

12

$$u = \frac{1}{\sqrt{3}} (1, -1, -1)$$

13

$$u = \frac{1}{\sqrt{266}} (-1, 11, -12)$$

14

$$u = \frac{1}{\sqrt{2}} (1, 0, -1)$$

15

$$\text{기울기벡터} : (6, 45), \text{방향미분계수} : -\frac{174}{\sqrt{17}}$$

16

$$\text{기울기벡터} : (2\sqrt{2}, 2, 2\sqrt{2}), \text{방향미분계수} : \frac{4}{3}\sqrt{2}$$

17

$$\text{기울기벡터} : \left(\frac{1}{3}, \frac{1}{2}, \frac{1}{6} \right), \text{방향미분계수} : \frac{1}{\sqrt{3}}$$

18

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기울기벡터 : $(12, 9)$, 방향미분계수 : $6\sqrt{3} + \frac{9}{2}$

19

기울기벡터 : $(1, 3, 0)$, 방향미분계수 : $\frac{43}{15}$

20

$$u = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$$

21

$$u = \left(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right)$$

22

$$u = \left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right)$$

23

$(2, 6)$

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SECTION 10.6 접평면과 선형근사

유제 10-22

접평면 $48x - 14y - z = 64$

법선벡터 $(48, -14, -1)$ (2021-07-21 updated)

유제 10-23

- 7

유제 10-24

증명 생략

유제 10-25

$$f(4,996,1.003) \approx \frac{5}{338} \frac{4996}{1000} + \frac{5}{18} \frac{1003}{1000} + \frac{1855}{3042}$$

유제 10-26

$$dz = (10xy^5 - 2)dx + (25x^2y^4 + 4)dy$$

10.6 연습문제

01

법선벡터 : $(14, -2, -1)$, 접평면 : $14x - 2y - z - 24 = 0$

02

법선벡터 : $(1, -1, -1)$, 접평면 : $x - y - z = 0$

03

법선벡터 : $(2, 0, 1)$, 접평면 : $2x + z + 2 = 0$

04

법선벡터 : $(0, -3, -1)$, 접평면 : $3y + z - 1 = 0$

05

법선벡터 : $(6, 0, -8)$, 접평면 : $3x - 4z + 50 = 0$

06

법선벡터 : $(1, 0, -2)$, 접평면 : $x - 2z = 0$

07

법선벡터: $(\frac{1}{4}, \frac{1}{6}, -1)$, 접평면 : $3x + 2y - 12z + 30 = 0$

08

법선벡터: $(-1, 0, -\pi)$, 접평면 : $x + \pi z - 2\pi = 0$

09

법선벡터 : $(8, 4, -1)$, 접평면 : $8x + 4y - z + 7 = 0$

10

법선벡터 : $(8, 8, -1)$, 접평면 : $8x + 8y - z - 9 - 8\ln 2 = 0$

11

증명 생략

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12

0.872

13

-0.0196

14

0.1009

15

0.03

16

$$dz = (10xy^5 - 2)dx + (25x^2y^4 + 4)dy$$

17

$$dz = (2\tan(x-3y)\sec^2(x-3y))dx + (-6\tan(x-3y)\sec^2(x-3y))dy$$

18

$$L = (1 + 3\ln 50) + (\ln 50 + \frac{9}{5})(x-3) + \frac{27}{50}(y-5)$$

19

$$L = -e^{\pi+2}(x-\pi)$$

20

$$L = \sqrt{45} + \frac{2}{\sqrt{45}}(x-2) + \frac{5}{\sqrt{45}}(y-5) + \frac{4}{\sqrt{45}}(z-4)$$

CHAPTER 10 다변수함수

SECTION 10.7 최댓값과 최솟값

유제 10-27

(a) $(15, -8)$

(b) $(0, 0), (6, 3), (-6, 3)$

유제 10-28

최솟값은 1, 최댓값은 28

10.7 연습문제

01

안장점 $(0, 0)$

02

안장점 $(0, 0)$

극솟값 -1

03

안장점 $(2, -2)$

04

안장점 $(4, -1)$

05

임계점 $(4, 2)$

극댓값 12

06

안장점 $(0, 0)$

극댓값 108

07

안장점 $(2, 1), (-2, 1)$

극솟값 0

08

안장점 $(0, 0)$

극솟값 1

09

극솟값 $\frac{4}{\sqrt{3}}$

10

안장점 $(1, -2), (-1, 2)$

극솟값 -14 , 극댓값 22

11

극솟값 $\frac{9}{2} - \frac{7}{2}\ln 2$

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임계점, 안장점 : $\left(-\frac{1}{3}, \ln 3\right)$

13

안장점 $\left(\frac{1}{2}, 1\right)$

14

안장점 $(4, 2), (-4, 2)$

극댓값 3

15

안장점 $(0, 0)$

극솟값 -27

16

최솟값 -1, 최댓값 30

17

최솟값 -11, 최댓값 1

18

최솟값 1, 최댓값 82

19

$x = y = z = 20$

20

$\frac{64}{3\sqrt{3}}r^3$

CHAPTER 10 다변수함수

SECTION 10.8 라그랑주 승수

유제 10-29

최댓값 $8\sqrt{3}$, 최솟값 $-8\sqrt{3}$

유제 10-30

최댓값 $\frac{2}{\sqrt{3}}$, 최솟값 $-\frac{2}{\sqrt{3}}$

유제 10-31

18

10.8 연습문제

01

최댓값 $\sqrt{2}$, 최솟값 $-\sqrt{2}$

02

최댓값 10, 최솟값 -6

03

최댓값 2, 최솟값 -1

04

최댓값 $6\sqrt{\frac{2}{3}} - 2$, 최솟값 $-6\sqrt{\frac{2}{3}} - 2$

05

최댓값 $2\sqrt{6}$, 최솟값 $-2\sqrt{6}$

06

최댓값 $4\sqrt{2}$, 최솟값 $-4\sqrt{2}$

07

최댓값 $\frac{8\sqrt{3}}{9}$, 최솟값 $-\frac{8\sqrt{3}}{9}$

08

$\left(-\frac{4}{5}, \frac{18}{5}\right)$

09

$\left(\frac{3}{29}, \frac{4}{29}, \frac{2}{29}\right)$

10

$\left(\frac{15}{14}, -\frac{11}{14}, \frac{8}{7}\right)$

11

$\left(\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}, 0\right), \left(-\sqrt{\frac{2}{3}}, -\sqrt{\frac{2}{3}}, 0\right)$

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12

$$\left(\frac{3}{\sqrt{5}}, \frac{6}{\sqrt{5}} \right) \text{ 또는 } \left(-\frac{3}{\sqrt{5}}, -\frac{6}{\sqrt{5}} \right)$$

13

최댓값 $3 + 2\sqrt{5}$, 최솟값 $3 - 2\sqrt{5}$

14

최댓값 3, 최솟값 1

15

$$\frac{51}{13}$$